



## Background noise estimation of geomagnetic signal

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**Abstract** A fast Fourier transform was applied to fit the geomagnetic diurnal variation. Fitting results showed that when the polynomial degree was greater than 160, the residual error was close to 0 nT. White noise is the main component of the residual error when the polynomial degree was greater than 160, so this method was adopted to calculate the background noise of the geomagnetic field. Spectrum analysis further demonstrated the necessity to remove background noise from geomagnetic data.

**Keywords:** background noise; geomagnetic field; spectrum analysis

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### 1. Introduction

The geomagnetic field is critical for solar activity monitoring, space weather detection, and some investigations into crustal motions that lead to geomagnetic field changes (Chapman et al., 1940; Campbell, 1997). Because of the various factors influencing the geomagnetic field, the sources of geomagnetic signal noise are also diverse. According to previous studies, the noise of the geomagnetic signal can be divided into two types (Ren, 2006). Variable noise changes over time, and comes from fluctuations within conductive fault zones or from instability within the observation environment, which acts as an antenna to couple with the external geomagnetic field. Background noise is more stable; it originates within a stable observation environment and from instrumental responses, such as thermal noise and other electronic noise. The noise within the geomagnetic data plays an important role in evaluating the quality of geomagnetic data and also has an impact on scientific research (Yao et al., 1995).

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Historically, variable noise in geomagnetic observations was calculated by first difference methods. In previous studies, some results such as temporal characteristics, spatial distribution features and influencing factors of variable noise were achieved (Yan, et al., 2013; Wang, et al., 2015). According to previous



35 research results, the intensity of variable noise is extremely weak and usually  
lower than an instrumental resolution of 0.1 nT. This suggests that background  
noise may be the main component of geomagnetic noise. However, previous  
research also showed that it is difficult to calculate background noise  
quantitatively because of the inseparability of the geomagnetic signal and noise. In  
40 seismology, signal-to-noise ratio (SNR) estimation on noisy data is mainly  
obtained through the energy superposition method, spectrum analysis or power  
spectrum calculation (Zhang et al., 2009). Because of the nonstationarity of the  
geomagnetic field, these methods are not suitable for geomagnetic SNR estimation.  
Zhu (2012, 2013) and Wang (2015) et al. applied principal component analysis  
45 (PCA) to suppress noise in airborne electromagnetic data. Nevertheless, PCA  
analysis needs observation data in at least three groups at the same observatory,  
which is improbable for most geomagnetic observatories. Jiang (2013) used  
maximum likelihood estimation to calculate geomagnetic noise through multiple  
iterations. Many previous studies have suffered from deficiencies in testing.

50 In this paper, the original diurnal geomagnetic data were fitted through a fast  
Fourier transform (FFT) and then the residual error was obtained to estimate  
background noise within the geomagnetic data. In previous studies, researchers  
applied an FFT to geomagnetic diurnal variation (Han et al., 2009; Zhao et al.,  
2014; Stephan et al., 2015; Yamazaki et al., 2017). However, almost all of them  
55 focused on the Sq diurnal variation, so the polynomial degree is no more than 6. In  
general, when geomagnetic disturbances are absent, the first four solar harmonics  
are sufficient to capture most of the variability in a daily record of the  
geomagnetic field. We suggest analysis of geomagnetic diurnal variation by the  
use of FFT with degree of 250; the residual error may represent changes in the  
60 background noise. Testing showed that this approach to estimate the noise within  
geomagnetic data is effective.

## 2. Data processing



The FFT is the most widely used method of spectrum analysis. Any periodic  
 65 signal can be decomposed into several components such as first harmonics ( $T$ ),  
 second harmonics ( $T/2$ ), third harmonics ( $T/3$ ) and more (Cooley and Tukey, 1965)  
 through the FFT transform.

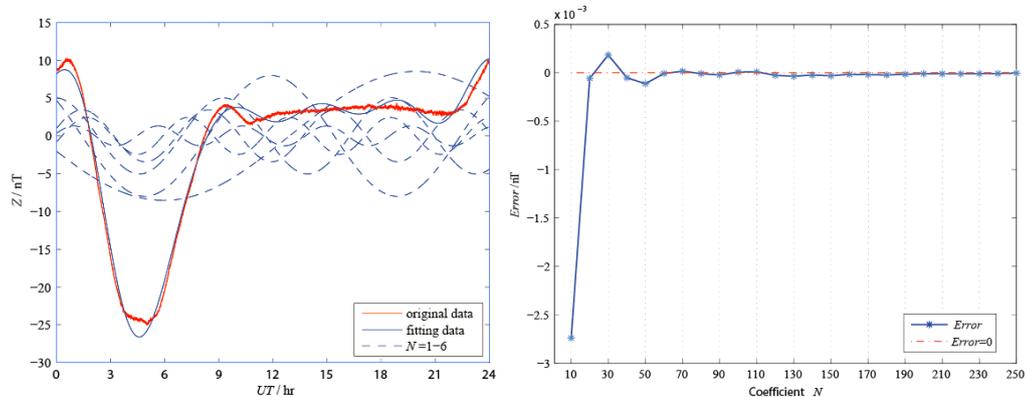
A time-series signal can be expressed as a function of sine and cosine as  
 follows:

$$70 \quad f(l) = \sum_{m=0}^{M-1} [a(m)\cos m\lambda_l + b(m)\sin m\lambda_l] \quad (1)$$

where  $a(m)$  and  $b(m)$  are coefficients of sine and cosine function, respectively.  
 $\lambda_l$  is a function of  $l$ , and  $l$  indicates the sequence number of the data series.  $M$   
 represents the total number of data point.

$$\lambda_l = \frac{2\pi}{M} l \quad (2)$$

75 Normal daily variation of the geomagnetic field mainly comprises the first  
 through sixth harmonic components (Fig. 1); these components represent signals  
 of period 24, 12, 8, 6, 4.8 and 4 hours, respectively: results of higher degree ( $N$ )  
 achieve closer fits.



80 Figure 1. Fitting results of first–sixth harmonic components      Figure 2. Error results of 10th–250th harmonic components

The background noise of the geomagnetic vertical component ( $Z$ ) is more  
 complex, because this component is more susceptible to variation in the  
 observation environment. To reduce the influence of the external geomagnetic



85 field, data from the quietest days of 2013 were chosen and the FFT transform was  
applied to fit them for 10–250 harmonic components. Residual error (*Error* in Eq.  
3) between original data and fitted spectrum was calculated by the following:

$$Error = \frac{\sum_{l=1}^M (ff(l) - f(l))}{M} \quad (l=1,2,3,\dots,86400) \quad (3)$$

Here,  $ff(l)$  represents the original geomagnetic data, and  $f(l)$  indicates the  
90 spectrum from the FFT. The sampling rate of the original data is 1 Hz, so the total  
number ( $M$ ) of samples in one day is 86400. As displayed in Fig. 2, residual error  
is less than 1.0 nT when the polynomial degree is more than 10 with smaller  
residual errors generally correlated with larger polynomial degrees. When the  
polynomial degree is greater than 160, the residual error approaches 0 nT. Based  
95 on previous analysis results, it is reasonable to assume that the spectrum of 160th  
degree could represent the original signal in which background noise is not  
contained.

Figure 3 shows the original signal, and the FFT-filtered spectrum with  
polynomial degree of 160, on 29 May 2013 at LYH observatory as an example; a  
100 constant is added between them for comparison. The filtered curve is almost  
identical to the original curve, though the original curve is smoother. This implies  
that the background noise of the geomagnetic signal could be estimated through  
FFT filtering with polynomial degree of 160. The background noise of the  
geomagnetic vertical component is marked as  $Z\_noise$  and is obtained from Eq.  
105 (4). Figure 4 shows the estimated background noise on 29 May 2013 at LYH  
observatory as an example. The background noise is randomly distributed between  
-0.2 and 0.2 nT with a mean value of 0 nT.

White noise is a random signal with a mean value of 0. Based on the  
characteristics of  $Z\_noise$  obtained from Fig. (4), we think the main composition  
110 of  $Z\_noise$  may be white noise, associated with the geomagnetic instrument and  
the observation environment.



$$Z\_noise = ff(l) - f(l) \quad (4)$$

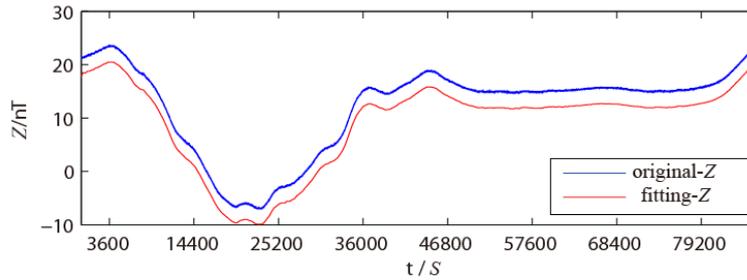


Figure 3. Original signal and fitting data of Z component

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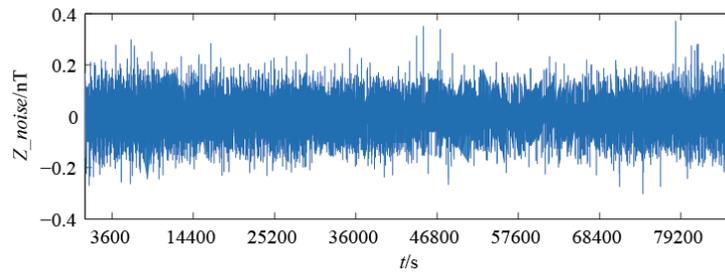


Figure 4. Estimated noise of Z component

### 3. Testing process

120 Standard white noise is a random signal with a mean value of 0, and its autocorrelation function is close to 0 when the lag ( $\tau$ ) is not equal to 0. To confirm that the main composition of  $Z\_noise$  is white noise, the autocorrelation function of  $Z\_noise$  is calculated from Eq. (5).

$$R_{xx} = \frac{1}{N} \sum_{t=0}^{N-1} x(t) \cdot x(t + \tau) \quad (5)$$

125  $R_{xx}$  is the autocorrelation function of signal,  $N$  represents the total number of data,  $x(t)$  indicates  $Z\_noise$ , and  $\tau$  is the lag. Figure 5 shows the autocorrelation function of the background noise on 29 May 2013 at LYH observatory. The autocorrelation function of  $Z\_noise$  clearly reaches up to 1 when  $\tau=0$  and which is close to 0 when  $\tau \neq 0$ , similar to the autocorrelation of white noise. Therefore, it is  
 130 demonstrated that the main composition of  $Z\_noise$  is white noise, and the



background noise of the Z component of the geomagnetic field ( $Z_{\text{noise}}$ ) could be obtained through FFT filtering with polynomial degree of 160.

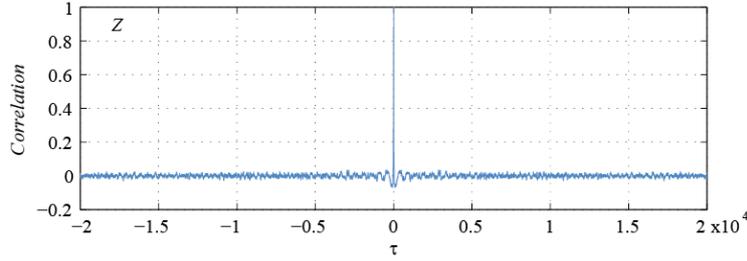


Figure 5. Autocorrelation function of background noise in Z component on 29 May 2013 at LYH

135 SNR of the geomagnetic signal can be calculated as Eq. (6) when background noise is obtained. The result shows that the SNR of the geomagnetic data from LYH observatory is about 47.

$$SNR_0 = 10 \cdot \log_{10} \left( \frac{\sum f^2(l)}{\sum (ff(l) - f(l))^2} \right) \quad (6)$$

140 **4. Spectrum analysis**

. To contrast the original geomagnetic data and the noise-free geomagnetic data, their frequency spectra were analyzed. Waveforms of the geomagnetic vertical component (Z) during each interval of 30 minutes were subjected to FFT spectrum analysis as follows:

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$$X(k) = \sum_{k=0}^{N-1} x(k) e^{-2\pi nki/N} \quad k = 0, 1, \dots, N-1 \quad (7)$$

(8)

Here,  $i = \sqrt{-1}$ ,  $R(u, v)$  and  $I(u, v)$  indicate the real and imaginary parts of the FFT result, respectively.  $F(u, v)$  represents the amplitude of the FFT spectrum.

150 The upper panel of Fig. 6 shows the amplitude spectrum of the original signal on 29 May 2013 and 30 May 2013 at LYH observatory. The amplitude spectrum of the original data is clearly more irregular. The background noise can be seen as scattered points in the spectrum, and the intensity of these points is not related to frequency or time. Because of the influence of background noise, the amplitude spectrum of the



original geomagnetic signal does not show any significant changes over this interval.  
155 The bottom panel of Fig. 6 represents the amplitude spectrum of the geomagnetic data  
after removing background noise. It is different from the upper panel. Irregularly  
scattered points representing background noises are removed almost completely and,  
as a result, the amplitude spectrum is more regular and informative. From the bottom  
panel of Fig. 6, some significant characteristics are obtained. First, geomagnetic  
160 energy changes with frequency: the lower the frequency of signal is, the higher the  
geomagnetic energy. Second, geomagnetic activity in each 30-minute block is  
different. In the first 24 half-hours of 29 May 2013 and the first 24 half-hours of 30  
May 2013 the geomagnetic activity is more intense than any others. In contrast, these  
characteristics are not immediately obvious in the amplitude spectrum of the original  
165 data, implying that the noise removing process is necessary for geomagnetic data  
analysis and that FFT filtering is an effective method for achieving this removal.

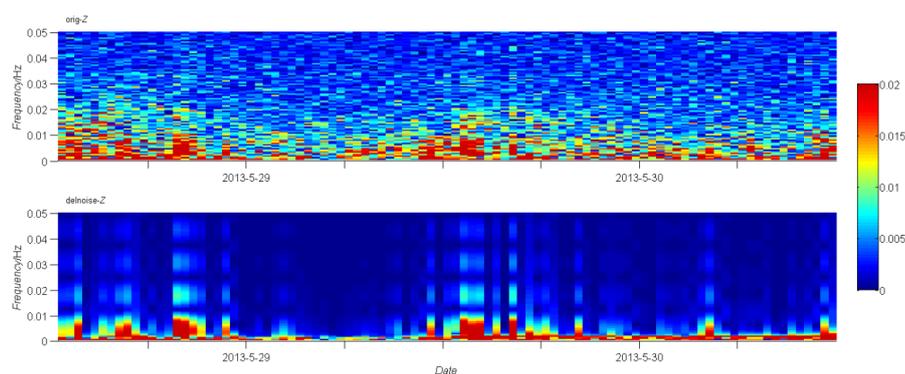


Figure 6 Spectrum of the original geomagnetic signal and the filtered data (Z component)

## 170 5. Conclusion and Discussion

The background noise of the geomagnetic signal can be obtained through FFT filtering with polynomial degree of 160 on the vertical component (Z) during the quietest days. The main conclusions are summarized as follows:

1. Residual error between the FFT-filtered data and the original approaches 0  
175 nT as the polynomial degree is greater than 160, and it has been confirmed



- that residual error with degree of 160 could represent the background noise of the geomagnetic signal.
2. Geomagnetic background noise is a random signal distributed between -0.2 and 0.2 nT.
  - 180 3. The autocorrelation function of background noise showed that it reaches up to 1.0 when lag ( $\tau$ ) is 0 and is close to 0 in other cases, which confirms that white noise is the main component of geomagnetic background noise.
  4. Spectrum analysis further confirms that it is necessary to remove the background noise of the geomagnetic signal, and that some geomagnetic  
185 changes are more remarkable after filtering.

FFT-filtered data with polynomial degree more than 160 could represent the original geomagnetic signal with period less than 540 s. Any signal with period less than 540 s in original data will be removed completely in filtered data. To avoid overprocessing, data from the quietest days were chosen, avoiding short-period  
190 variations such as pulsations or geomagnetic bays. In addition, because the vertical component (Z) contains more noise information than other components and is not as susceptible to the external geomagnetic field, it was chosen as the analysis object in this paper. Because the main factors influencing background noise are local observation environment and instrumental response, the geomagnetic background  
195 noise at different observatories differs. For one certain observatory, background noise is nearly invariable due to the stability of observation environment and instrument condition usually.

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